Global solvability for a class of complex vector fields on the cylinder

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Abstract:

Let $\mathcal{L}$ be a nonsingular complex vector field defined on a smooth, paracompact, noncompact, two-dimensional manifold Ω. Assume that $\mathcal{L}$ satisfies the *Nirenberg-Treves* condition $(\mathcal{P})$.

We say that $\mathcal{L}$ is *globally solvable* if

$$\mathcal{L} : C^\infty(\Omega) \to C^\infty(\Omega)$$

has closed image; if, moreover, $\dim(\ker {}^t\mathcal{L}) < \infty$ then we say that $\mathcal{L}$ is *strongly solvable*.

It is known that condition $(\mathcal{P})$ is necessary to strong solvability.

It was proved by Hounie (extending results due to Malgrange and Hörmander) that if

$(\sharp)$ no orbit is relatively compact in Ω.

then $\mathcal{L}C^\infty(\Omega) = C^\infty(\Omega)$; consequently, $\mathcal{L}$ is strongly solvable.

In this lecture we will deal with strong solvability of a class of complex vector fields defined on the cylinder $\Omega = \mathbb{R} \times S^1$, for which $(\sharp)$ fails.

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