# II International Workshop
# Global Properties of PDE's on Manifolds

September 17-19, 2014

Department of Mathematics and Informatics
University of Cagliari

# Abstracts

### Wednesday, September 17

JOACHIM TOFT
Linnæus University, Sweden

**Modulation spaces, harmonic analysis and pseudo-differential operators.**

**Abstract**: In the present talk we present recent results on composition, continuity and Schatten-von Neumann (SvN) properties for operators and pseudo-differential operators ($\Psi$DOs) when acting on modulation spaces. For example we present necessary and sufficient conditions in order for the Weyl product should be continuous on modulation spaces. Such question is strongly connected to questions wether compositions of $\Psi$DOs with symbols in modulation spaces remain as $\Psi$DOs with a symbol in a modulation space.

We also present necessary and sufficient conditions for $\Psi$DOs with symbols in modulation spaces should be SvN operators of certain degree in the interval $(0, \infty]$. Note here that there are so far only few results in the literature on SvN operators with degrees less than one.

The talk is based on joint works with K. H. Gröchenig, and with E. Cordero and P. Wahlberg.

*E-mail address: joachim.toft@lnu.se*

STEVAN PILIPOVIĆ
University of Novi Sad, Serbia

**Linear and semilinear pseudodifferential equations in spaces of tempered ultradistributions.**

**Abstract**: We study a class of linear and semilinear elliptic equations on spaces of tempered ultradistributions of Beurling and Roumieu type. Assuming that the linear part of the equation is a pseudodifferential operator of infinite order satisfying a suitable ellipticity condition we prove a regularity result in the functional setting above for weak Sobolev type solutions.

This talk is based on the joint papers with Marco Cappiello and Bojan Prangoski.

*E-mail address: pilipovic@dmi.uns.ac.rs*

MARCO CAPPIELLO
Università di Torino, Italy

**On the propagation of the analytic regularity for semilinear hyperbolic systems in $\mathbb{R}^d$.**

**Abstract**: We study the propagation of analytic regularity for semilinear symmetric hyperbolic systems on $\mathbb{R}^d$. Namely we prove that if the initial datum extends to a holomorphic function in a strip of radius (=width) $\epsilon_0$, the same happens for the solution $u(t, \cdot)$ for a certain radius $\epsilon(t)$, as long as the solution exists.

Our approach allows to give precise lower bounds on the spatial radius of analyticity $\epsilon(t)$ as $t$ grows and applies also to the Schrödinger equation with a real-analytic electromagnetic potential and to the Euler equations.

The results above have been obtained in collaboration with Fabio Nicola (Politecnico di Torino) and Piero D'Ancona (Sapienza Università di Roma).

*E-mail address: marco.cappiello@unito.it*

## Thursday, September 18

TATSUO NISHITANI
Osaka University, Japan

**Weyl-Hörmander calculus and the Cauchy problem in the Gevrey classes.**

**Abstract**: We discuss the classical Bronshtein's result about the well-posedness of the Cauchy problem in the Gevrey classes within a framework of Weyl-Hörmander calculus. We transform the reference operator to another one by a conjugation of an exponential weight which is independent of the dual variable of the time. Using the Lipschitz continuity of the characteristic roots, we prove that thus transformed operator has an inverse within a framework of Weyl-Hörmander calculus in the whole space with a metric which is independent of both space and the dual of the time variables.

Let $P(x, \xi)$ be a hyperbolic operator of order $m$ with respect to $x_1$ with coefficients in the Gevrey class $m/(m-1)$ where $x = (x_1, ..., x_n)$ and $\xi = (\xi_1, \xi') = (\xi_1, \xi_2, ..., \xi_n)$. Choosing a smooth $\rho(x_1; \epsilon)$ with a parameter $\epsilon > 0$ such that $\rho(x_1; \epsilon) > 0$, $\rho'(x_1; \epsilon) < 0$ suitably we consider the conjugate operator

$$\tilde{P}(x, D) = \mathrm{Op}(e^{\rho(x_1)\langle \xi' \rangle_\gamma^\kappa}) P(x, D) \mathrm{Op}(e^{-\rho(x_1)\langle \xi' \rangle_\gamma^\kappa})$$

where $\kappa = (m-1)/m$ and $\langle \xi' \rangle_\gamma = (\gamma^2 + |\xi'|^2)^{1/2}$. Let $g = \epsilon \langle \xi' \rangle_\gamma^{2(1-\kappa)} |dx|^2 + \epsilon \langle \xi' \rangle_\gamma^{-2\kappa} |d\xi|^2$ then we have

**Theorem 1** *$g$ is an admissible metric on $\mathbb{R}^n \times \mathbb{R}^n$. If $\gamma \geq \gamma_0(\epsilon)$ then $|\tilde{P}(x, \xi)|^{\pm 1}$ are $g$ admissible weights and we have*

$$\tilde{P}(x, \xi)^{\pm 1} \in S(|\tilde{P}(x, \xi)|^{\pm 1}, g).$$

*Taking $\epsilon > 0$ small there exist $Q_i(x, \xi) \in S(1, g)$ such that*

$$\tilde{P}(x, \xi) \# Q_1(x, \xi) = 1, \quad Q_2(x, \xi) \# \tilde{P}(x, \xi) = 1.$$

*E-mail address: nishitani@math.sci.osaka-u.ac.jp*

JEAN VAILLANT
UPMC Paris 6, France
**Conditions of hyperbolicity of linear differential systems.**

**Abstract**: Let $x = (x_0, x') = (x_0, x_1, \ldots, x_n) \in \Omega$, $\Omega$ neighbourhood of $0 \in \mathbb{R}^{n+1}$, we consider an $N \times N$ linear first order system of differential operators

$$h(x, D) = a(x, D) + b(x)$$

where $D = (D_0, D') = (D_0, D_1, \ldots, D_n)$, $D_0 = \frac{\partial}{\partial x_0}$, $D_j = \frac{\partial}{\partial x_j}$. $a(x, \xi)$ is the principal symbol of $h$, $\xi = (\xi_0, \xi') = (\xi_0, \xi_1, \ldots, \xi_n)$. $a$ and $b$ are $N \times N$ matrices.

We consider the Cauchy problem for $h$:

$$\begin{cases} h(x, D)u(x) = f(x)\,, \\ u|_{x_0 = \underline{x_0}} = u_0(x')\,. \end{cases} \tag{1}$$

**Definition 1** *$h$ is hyperbolic means that the Cauchy problem 1 is uniformly well-posed in $\mathcal{C}^\infty(\Omega)$.*

We assume that $\det a(x; \xi)$ is a hyperbolic polynomial of *constant multiplicity*:

We have previously defined the conditions $L$ - *Vaillant-1991*.

We intend to state that the conditions $L$ are necessary and sufficient in order that the operator is hyperbolic.

We introduce a suitable tool (cf. Berzin Vaillant).:$h$ is *diagonalizable with a good decomposition*

In a precedent paper (2014 Journal Fixed Point)we have prove that the conditions $L$ are sufficient for the well-posedness . The necessity has been stated up to multiplicity $\leq 5$ in *Vaillant-1999*. In this lecture we begin the study of the necessity in a general case.

*E-mail address: jean.vaillant@upmc.fr*

PETAR POPIVANOV
IMI - BAS, Bulgaria
**Exact solutions of several equations of Mathematical physics and differential geometry.**

**Abstract**: This talk deals with exact solutions of several equations of Mathematical physics as the multidimensional semilinear Klein Gordon equation, Szego equation and others.

It is proposed locally a full description of the solutions of some nonlinear elliptic systems of PDE, arising in geometry.

*E-mail address: popivano@math.bas.bg*

TODOR GRAMCHEV
Università di Cagliari, Italy

**Global Hypoellipticity for First Order Pseudo-differential Operators on Compact Manifolds.**

**Abstract**: We study the global hypoellipticity of first order operators of type

$$L = D_t + a(t)Q(x, D) + ib(t)P(x, D), \quad D_t = i^{-1}\partial_t, \tag{2}$$

where $(t, x) \in \mathbb{T} \times M$, $a, b$ are real smooth functions on $\mathbb{T}$, and $P(x, D), Q(x, D)$ are self-adjoint first order pseudo-differential operators, defined on a closed n-dimensional smooth Riemannian manifold $M$. We assume that there exists an elliptic normal pseudo–diffferential operator $E(x, D)$ of order $m > 0$, satisfying the commutator hypotheses:

$$[E, P(x, D)] = [E, Q(x, D)] = 0. \tag{3}$$

We show that under the additional assumption

$$[P(x, D), Q(x, D)] = 0 \tag{4}$$

we are able to derive necessary and sufficient conditions on the operators $P$ and $Q$ for the global hypoellipticity of $L$.

The results are obtained in collaboration with F. Avila and A. Kirilov (Univ. of Paraná - Brazil).

*E-mail address: todor@unica.it*

FERNANDO ÁVILA
Universidade Federal do Paraná, Brazil and Università di Cagliari, Italy

**Global Hypoellipticity for First P.D.O's with Logarithmic Perturbations on Compact Manifolds.**

**Abstract**: On the study of global hypoellipticity of operator $L$ in (2), a fundamental ingredients of our approach are the use the *Weyl's Asymptotic Formula*:

$$\lambda_j \sim c_0 j^{\frac{m}{n}}, \quad j \longrightarrow +\infty,$$

where $\sigma(E(x, D)) = \{0 < \lambda_1 \leq \ldots \leq \lambda_j \to \infty\}$ is the spectrum of $E(x, D)$, and the discrete characterization of $H^s(\mathbb{T} \times M)$ and $C^\infty(\mathbb{T} \times M)$ by the eigenfunction expansions defined by $E(x, D)$.

The commutation (3) allows us to reduce the study the regularity of solutions $u \in \mathcal{D}'(\mathbb{T} \times M)$ of the equation $Lu = f \in C^\infty(\mathbb{T} \times M)$ to the study sequence of scalar linear ODE on $\mathbb{T}$.

Moreover, if $\{\varphi_j(x)\}_{j\in\mathbb{N}}$ is an orthonormal basis of $L^2(M)$, associated to $\sigma(E)$, by (3) and admitting E(x,D) has only simple eigenvalues, one can find real sequences $\mu_j$ and $\nu_j$ such that

$$P(x, D)\varphi_j(x) = \mu_j\varphi_j(x) \text{ and } Q(x, D)\varphi_j(x) = \nu_j\varphi_j(x), \ j \in \mathbb{N}.$$

Thus if $|\nu_j| \to \infty$, the global hypoellipticity of $L$ depends of the following expressions:

$$\limsup_{j\to\infty} \frac{|\nu_j|}{\log(j)} \quad \text{and} \quad \inf_{\ell\in\mathbb{Z}} |a_0\mu_j + \ell|,$$

where $a_0 = (2\pi)^{-1}\int_0^{2\pi} a(t)dt$.

*E-mail address: favilasi@gmail.com*

MONICA MARRAS
Università of Cagliari, Italy

**Reaction-diffusion problems under various boundary conditions with blow-up solutions.**

**Abstract**: The question of blow-up of solutions to nonlinear parabolic equations and systems has received considerable attention in the recent literature. In practical situations one would like to know among other things whether the solution blows up and, if so, at which time blow-up occurs.

When the solution does blow up at some finite time T, this time can seldom be determined explicitly, so much effort has been devoted to the calculation of bounds for T. Most of the methods used until recently have yielded only upper bounds for T, so that in particular problems in which blow-up has to be avoided, they are of little value.

We investigate the question of blow-up for nonnegative classical solutions of some nonlinear parabolic problems defined in a bounded domain. Under conditions on data and geometry of the spatial domain, explicit upper and lower bounds for the blow-up time are derived.

*E-mail address: mmarras@unica.it*

ALEXANDER LECKE
University of Vienna, Austria

**Hermite expansions for Gelfand-Shilov type spaces.**

**Abstract**: This talk is due to a joint work with Prof. T. Gramchev, L. Rodino and S. Pilipović.
Gelfand-Shilov spaces $\mathcal{S}_\alpha(\mathbb{R}^d), \mathcal{S}^\beta(\mathbb{R}^d)$ and $\mathcal{S}_\alpha^\beta(\mathbb{R}^d)$ and their generalisations, the Gelfand-Shilov spaces of Roumieau and Beuerling type $\mathcal{S}^{\{m\}}(\mathbb{R}^d)$ respectivly $\mathcal{S}^{(m)}(\mathbb{R}^d)$ are frequently discussed. In this talk we focus on the special cases $\mathcal{S}_\alpha^\beta(\mathbb{R}^d)$ (resp. $\sum_\alpha^\beta(\mathbb{R}^d)$). We show that if it converges in the sense of $\mathcal{S}_\alpha^\beta(\mathbb{R}^d)$ (resp. $\sum_\alpha^\beta(\mathbb{R}^d)$), then it belongs to $\mathcal{S}_\alpha^\alpha(\mathbb{R}^d)$ (resp. $\sum_\alpha^\alpha(\mathbb{R}^d)$), i.e. for a function $f = \sum a_k\Psi_k \in \mathcal{S}_\beta^\beta(\mathbb{R}^d)$ (resp. $\sum_\beta^\beta(\mathbb{R}^d)$) in order that $\sum a_k\Psi_k \to f$ in $\mathcal{S}_\alpha^\beta(\mathbb{R}^d)$ (resp. $\sum_\alpha^\beta(\mathbb{R}^d)$), where $\frac{1}{2} \le \alpha \le \beta$ (resp. $1/2 < \alpha \le \beta$), it follows that $\alpha = \beta$.
Furthermore we analyze intermideate spaces $\big(\mathcal{S}_\alpha^\alpha \otimes \mathcal{S}_\beta^\beta\big)(\mathbb{R}^{s+t})$ (resp. $\big(\Sigma_\alpha^\alpha \otimes$

$\Sigma_{\beta}^{\beta})(\mathbb{R}^{s+t})$), introduced also by Gelfand and Shilov, through the estimates of Hermite coefficients. The elements of spaces of this type are functions $f$ which behave in their first $s$ components like a function in $\mathcal{S}_{\alpha}^{\alpha}(\mathbb{R}^{s})$ (resp. $\sum_{\alpha}^{\alpha}(\mathbb{R}^{s})$) and in their last $t$ components like a function in $\mathcal{S}_{\beta}^{\beta}(\mathbb{R}^{t})$ (resp. $\sum_{\beta}^{\beta}(\mathbb{R}^{t})$). In the last part of the talk we introduce one more class of Gelfand-Shilov type spaces $S_{\sigma}^{\otimes,\sigma}(\mathbb{R}^{n}), \sigma \geq 1/2$, and $\Sigma_{\sigma}^{\otimes,\sigma}(\mathbb{R}^{n}), \sigma > 1/2$. These spaces were obtained through the iteration of Harmonic oscilators and are related to our study of Weyl formula for tensorised products of elliptic Shubin type operators. We compare all the considered spaces through the estimates of Hermite coefficients.

*E-mail address: alexander.lecke@univie.ac.at*

MARIA A. FARINA
Università di Cagliari, Italy

**Optimization of the principal eigenvalue under mixed boundary conditions.**

**Abstract**: We investigate biologically-oriented problems, motivated by the question of determining the most convenient spatial arrangement of favorable and unfavorable resources for a species to survive or to decline.

We prove existence and uniqueness results, and present some features of optimizers.

*E-mail address: mafarina@unica.it*

GIORGIA TRANQUILLI
Università di Cagliari, Italy

**Hyperbolic Cauchy problems for second order Shubin pseudo-differential operators.**

**Abstract**: We investigate the global well-posedness in $\mathbb{R}^{n}$ in scales of weighted Sobolev–Shubin spaces and in the Gelfand–Shilov classes $S_{\mu}^{\mu}(\mathbb{R}^{n})$ of the Cauchy problem in $\mathbb{R}^{n}$ for some wave equations associated to second order globally elliptic Shubin pseudodifferential operators with principal real part. We also study a case of non globally elliptic operator represented by the twisted Laplacian.

The talk is based on joint work with T. Gramchev (Università di Cagliari).

*E-mail address: tranquilli@unica.it*