(A, B, C) formulas for some evolution equations.

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Abstract

Many nonlinear differential equations can be solved via the Inverse Scattering Transform (IST). In this talk, after a brief introduction of the IST, we derive an explicit solution formula for two interesting evolution equations, the sine-Gordon equation and the matrix focusing nonlinear Schrödinger equation (mNLS). In order to obtain these solutions we use a triplet of constant matrices (A, B, C) and the matrix exponential. In particular, the kernels of the corresponding Marchenko integral equations are expressed in terms of the triplet (A, B, C), where the constant matrix A is $p \times p$ with eigenvalues having positive real parts and B and C are rectangular constant matrices of dimensions $p \times n$ and $m \times p$, respectively, with $m = n = 1$ for the sine-Gordon equation. Since the kernels constructed using the triplet (A, B, C) are separable, we get the explicit solutions of the Marchenko integral equations which yield the exact solutions of the evolution equations mentioned above. More precisely, starting with any matrix triplet (A, B, C) such exact solutions are constructed in terms of the unique solutions Q and N to the respective Lyapunov equations $A^*Q + QA = C^*C$ and $AN + NA^* = BB^*$ with the asterisk denoting the matrix conjugate transpose. The solution formulas presented yield already known exact solutions (breathers, solitons, multi-solitons, etc.) for special choices of the matrix triplet (A, B, C) in concise form.

This talk is based on joint work with T. Aktosun and T. Busse of the University of Texas at Arlington and C. van der Mee of the University of Cagliari. Research supported by the Autonomous Region of Sardinia under grant PO Sardegna 2007-2013, L.R. 7/2007, and by INdAM-GNFM.