MINIMAL SURFACES IN S^3 FOLIATED BY CIRCLES

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We deal with minimal surfaces in the unit sphere S^3, which are one-parameter families of circles. Minimal surfaces in $\mathbb{R}^3$ foliated by circles were first investigated by Riemann, and a hundred years later Lawson constructed examples of such surfaces in S^3. We prove that in S^3 there are only two types of minimal surfaces foliated by circles, crossing the principal lines at a constant angle. The first type surfaces are foliated by great circles, which are bisectrices of the principal lines, and we show that these minimal surfaces are the well-known examples of Lawson. The second type surfaces, which are new in the literature, are families of small circles, and the circles are principal lines. We give a constructive formula for these surfaces.

We point out the relation between the theory of minimal surfaces in S^3 and the theory of minimal foliated semi-symmetric hypersurfaces in $\mathbb{R}^4$. Each minimal surface in S^3 generates a minimal foliated semi-symmetric hypersurface in $\mathbb{R}^4$ according to a special construction given by G. Ganchev and V. Milousheva. We illustrate how this construction can be applied to two examples of minimal surfaces in S^3 for obtaining the first and the second type helicoids, which are special minimal foliated semi-symmetric hypersurfaces. We also apply the construction to a class of generalized tori of second type and thus we obtain new minimal foliated semi-symmetric hypersurfaces in $\mathbb{R}^4$.