$\sigma-$EVOLUTION MODELS WITH LOW REGULAR TIME-DEPENDENT STRUCTURAL DAMPING: NON-EFFECTIVE DISSIPATION

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Resumo

We consider, for $0 < \theta \leq \sigma$, the initial value problem for a $\sigma-$evolution equation with fractional damping in $\mathbb{R}^n$:

$$u_{tt}(t,x) + A^\sigma u(t,x) + b(t)A^\theta u_t(t,x) = 0, \qquad (t,x) \in (0,\infty) \times \mathbb{R}^n \tag{1}$$

with initial data

$$u(0,x) = u_0(x), \quad u_t(0,x) = u_1(x), \qquad x \in \mathbb{R}^n, \tag{2}$$

where $A := -\Delta = -\sum_{i=1}^{n} \frac{\partial^2}{\partial x_i^2}\cdot$

The fractional power operator $A^\delta : \mathcal{D}(A^\delta) \subset L^2(\mathbb{R}^n) \to L^2(\mathbb{R}^n)$ $(\delta \geq 0)$ with its domain $\mathcal{D}(A^\delta) = H^{2\delta}(\mathbb{R}^n)$ is defined by

$$A^\delta v(x) := \mathcal{F}^{-1}\big(|\xi|^{2\delta}\mathcal{F}(v)(\xi)\big)(x), \qquad v \in H^{2\delta}(\mathbb{R}^n), \qquad x \in \mathbb{R}^n,$$

where $\mathcal{F}$ denotes the usual Fourier transform in $L^2(\mathbb{R}^n)$ and $|\cdot|$ denotes the usual norm in $\mathbb{R}^n$. The results obtained in this work can be applied to several initial value problems associated to second-order equations, as for example, wave equation, plate equation, among others.

We assume, for a sufficient large $t_0 > 0$, that $b \sim g$ in $[t_0, \infty)$, in other words, there exist $a_1 > 0$ and $a_2 > 0$ such that $a_1 g(t) \leq b(t) \leq a_2 g(t)$ for all $t \geq t_0$, in which $g(t) = (1+t)^\alpha ln^\gamma(1+t)$. Furthermore, we assume the following hypothesis:

Hypothesis A: For $0 < \theta \leq \sigma$, let $\alpha \in [-1, 1)$ and $\gamma \in \mathbb{R}$ satisfying one of the following:

(i) $\alpha \in (-1,1)$ and $\sigma(1+\alpha) < 2\theta$;

(ii) $\alpha \in (-1,1)$, $\gamma \leq 0$, and $\sigma(1+\alpha) = 2\theta$;

(iii) $\alpha = -1$ and $\gamma \geq -1$.

The asymptotic profile of (1)-(2) for $\sigma > 0$, $\theta \in (0,\sigma)$, $b(t) = 2\mu(1+t)^\alpha$, $\mu > 0$ and $\alpha \in (-1,1)$, was investigated by D'Abbicco-Ebert in [2]. They proved an anomalous diffusion phenomena for this equation and introduced a classification based on it: the damping is said *effective* when the diffusion phenomenon holds and *non-effective* otherwise. This concept generalized the classification introduced by J. Wirth for $\theta = 0$ in [10] (non-effective case) and [5] (effective case). Furthermore, D'Abbicco-Ebert reported that when $2\theta < \sigma(1+\alpha)$ the damping is effective and non-effective if $2\theta > \sigma(1+\alpha)$. The case $2\theta = \sigma(1+\alpha)$ is treated as a critical case and they do not discuss.

Going back to Hypothesis A and based on the last classification introduced, with except for the case $\sigma(1+\alpha) = 2\theta$ and $\gamma = 0$ (critical instance), we have exactly the non-effective damping case. The effective case will be treated in a forthcoming paper [9]. Our classification however, is not motivated by whether the asymptotic profile of the solution of the problem has or not an diffusion phenomena, but rely on a new classification in which will be introduced next. The connection between our new classification and the diffusion phenomena is a open question.

The objective of this work is to develop a method to obtain optimal decay rates $L^p - L^q$ for the solution of (1)-(2), with $1 \leq p \leq 2 \leq q \leq \infty$, considering only $b(t) \sim (1+t)^\alpha ln^\gamma(1+t) =: g(t)$, that is, no control in $\frac{d}{dt}b$ will be assumed. Furthermore, it should be noticed that this work can be extended for a more general class of functions g. In addition, our method can be applied to other equations, for example, equations that include a rotational inertia term.

To develop our method, we back to the origin of the energy method in Fourier space but at the same time considering the knowledge provided by the diagonalization procedure and the method due to Charão-da Luz-Ikehata [1] and [4]. For this sake, we consider hyperbolic and elliptic zones *similarly* as considered in the diagonalization procedure, see for example [7] and [8]. In the diagonalization procedure the zones came from WKB analysis, in our case the zones comes together with a energy multiplier, that is, comes from an algebraic understand of the problem. For each ξ such that $0 < |\xi| \leq R$, we consider $\psi(\xi) := |\xi|^{2\theta} g(t_\xi)$, in which t_ξ separates low zone from elliptic and hyperbolic zones. We introduce a new classification based on the comparison between $|\xi|^\sigma$ and $\psi(\xi)$. The aim of this paper is to investigate the case $\max\{|\xi|^\sigma, \psi(\xi)\} = |\xi|^\sigma$ for small frequency in which correspond to our assumptions made on θ, σ, α and γ in Hypothesis A. The remaining case requires an improvement of the energy method and will be treated in a forthcoming paper [9].

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